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JOURNAL DE PHYSIQUE
Colloque C6, supplément au n°6, Tome 45, juin 1984 page C6-143

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Résumé - Les effets de corrélation sont inclus dans le calcul des courants dans un noyau tournant par l'intermédiaire de la densité à une particule. La correction à la densité due au potentiel tournant peut être divisée en deux parties : la première ne dépend que du champ de pairing constant Δ mais donne un champ de courant nul dans le laboratoire pour les grandes valeurs de Δ . La deuxième partie dépend de la réaction de Δ au potentiel tournant et donne, comme attendu, un courant irrotationnel.

Abstract - The pairing effects are included in the calculation of the currents in a rotating nucleus by means of the single particle density. The correction to the density due to the rotating field may be shared into two parts :the first one depends only on the constant pairing field Δ but leads to no current in the laboratory frame for high Δ values. The second part depends on the reaction of the pairing field to the rotational field and leads to the expected irrotationnal current flow.

I - INTRODUCTION

The importance of pairing correlations has been pointed out in the calculation of the moments of inertia for collective rotations.
It is well known that the Inglis cranking formula gives a value very close to the rigid body moment of inertia and thus about twice the experimental value. On the other hand, the value for an irrotational fluid is too small ($\mathcal{J}_{irrot} < \mathcal{J}_{exp} < \mathcal{J}_{Inglis}$). The introduction of superfluidity in the Inglis formula by Belyaev [1] made it possible to correct the previous cranking results and to approach the experimental values [2]. Thus, it is clear that the pairing correlations are necessary to account for the moment inertia:

$$\mathcal{J} = \frac{\langle l_x \rangle}{\omega} \tag{1}$$

for a nucleus rotating with the angular velocity ω around the x-axis.

The moment of inertia being a mean value, the conclusion is that the pairing correlations must be included in the density matrix ρ.
Since the current in a rotating nucleus is :

$$\vec{j} = \rho \vec{v} \tag{2}$$

with the same density ρ as for the moment of inertia, the usual currents must be corrected by the pairing effects as well. In fact, $\mathcal{J}$ and $\vec{j}$ are not independent

$$\mathcal{J} = \frac{\vec{\omega}}{\omega^2}\int d^3\vec{r}\ \vec{r}\wedge m\vec{v}(\vec{r})\ \rho(\vec{r}) = \frac{\vec{\omega}}{\omega^2}\int d^3\vec{r}\ \vec{r}\wedge\vec{j}(\vec{r}) \tag{3}$$

and the dependance of $\mathcal{J}$ on the pairing field is correlated to the dependance of $\vec{j}$ on the same field.
The aim of this work is to evaluate the pairing correlation effects on the flow patterns in a semiclassical way, and for a rotating anisotropic harmonic oscillator.
For such a potential it has been shown that, without pairing : [4)]
- quantum mechanical calculations exhibit vortices in the flow
- on the contrary, semiclassical calculations give a very smooth flow, looking like a rigid body rotation
- at the limit of very heavy nuclei ($A\to\infty$) the flow tends towards a rigid body one in agreement with the semiclassical results [5)]. Thus, in semiclassical calculations including pairing, no other shell effects may occur and the occurence of any vortices will just be due to the superfluidity of the system and not to any other quantal effect. The semiclassical method turns out to be a good tool to investigate the pairing correlation effects.

II - CONTRIBUTION OF THE PAIRING FIELD TO THE ONE BODY NORMAL DENSITY OF A ROTATING NUCLEUS.

Following on from Migdal [6)], for a system of quasiparticles described by the Hamiltonian $\mathcal{H} = H + V$, where V is a weak perturbation, with

$$H\varphi_\lambda = e_\lambda \varphi_\lambda$$

$$\varepsilon_\lambda = e_\lambda - \mu_F \qquad \mu_F = \text{Fermi energy}$$

$$E_\lambda = \sqrt{\varepsilon_\lambda^2 + \Delta^2} \qquad \text{usual quasi-particle energy}$$

$$\Delta = \text{constant} \qquad \text{same pairing field for all states}$$

the density is given by :

$$\rho_{\lambda\lambda'} = \frac{E_\lambda - \varepsilon_\lambda}{2E_\lambda}\,\delta_{\lambda\lambda'} + \rho'_{\lambda\lambda'}$$

where the second term $\rho'_{\lambda\lambda'}$ is the correction due to the perturbative potential :

$$\rho'_{\lambda\lambda'} = \frac{(\varepsilon_\lambda\varepsilon_{\lambda'} - E_\lambda E_{\lambda'})V_{\lambda\lambda'} - \Delta^2 V^*_{\lambda\lambda'}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} + \Delta\,\frac{\varepsilon_\lambda \Delta'_{\lambda\lambda'} + \varepsilon_{\lambda'}\Delta'^*_{\lambda\lambda'}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} \qquad (4)$$

$$= \rho^{(1)}_{\lambda\lambda'} + \rho^{(2)}_{\lambda\lambda'}$$

- $\rho^{(1)}_{\lambda\lambda'}$ is the change due to the perturbation without any change in the pairing field : it is the usual Belyaev contribution
- $\Delta'_{\lambda\lambda'}$ is the change in the pairing field due to its reaction to the perturbative potential V and then $\rho^{(2)}_{\lambda\lambda'}$ is the corresponding response of the density. $\Delta'_{\lambda\lambda'}$ obeys the following equation

$$\sum_{\lambda\lambda'} \frac{2\Delta(\varepsilon_\lambda V^*_{\lambda\lambda'} + \varepsilon_{\lambda'}V_{\lambda\lambda'}) + 2\Delta^2\Delta'_{\lambda\lambda'} + [2\Delta^2 + (\varepsilon_\lambda - \varepsilon_{\lambda'})^2]\Delta'^*_{\lambda\lambda'}}{E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})}\,\varphi_\lambda(\vec{r})\varphi^*_{\lambda'}(\vec{r}) = 0 \qquad (5)$$

In the case of a nucleus rotating slowly around the x-axis, with the angular velocity ω , the perturbative potential is :

$$V = -\omega \ell_x = -V^*$$

(ℓ_x = x-component of the angular momentum) in the nucleus-fixed system of coordinates.
V being purely imaginary and proportionnal to ω , the reaction Δ' to this potential may be written 6) :

$$\Delta'(\vec{r}) = i\omega \, f(\vec{r}) \tag{6}$$

Eqs (4) and (5) now read :

$$\rho^{(1)}_{\lambda\lambda'} = \omega \, \frac{E_\lambda E_{\lambda'} - \varepsilon_\lambda \varepsilon_{\lambda'} - \Delta^2}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} \, \ell_{\lambda\lambda'} \tag{7.1}$$

$$\rho^{(2)}_{\lambda\lambda'} = i\omega\Delta \, \frac{\varepsilon_\lambda - \varepsilon_{\lambda'}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} \, f_{\lambda\lambda'} \tag{7.2}$$

$$\sum_{\lambda\lambda'} \frac{2\Delta\hbar \dot{\ell}_{\lambda\lambda'} + f_{\lambda\lambda'}(\varepsilon_\lambda - \varepsilon_{\lambda'})^2}{E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} \varphi_\lambda(\vec{r})\varphi^*_{\lambda'}(\vec{r}) = 0 \tag{8}$$

(the subscript x in ℓ is omitted because there is no possible confusion)
The function $[E_{\lambda'}E_\lambda(E_\lambda + E_{\lambda'})]^{-1}$ has a sharp maximum at $\varepsilon_\lambda + \varepsilon_{\lambda'} = 0$ for a fixed value of $(\varepsilon_\lambda - \varepsilon_{\lambda'})$ and, according to Migdal 6):

$$[E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})]^{-1} \sim \frac{1}{\Delta^2} \, g\left(\frac{\varepsilon_\lambda - \varepsilon_{\lambda'}}{2\Delta}\right)\delta(\varepsilon_\lambda + \varepsilon_{\lambda'})$$

$$\frac{E_\lambda E_{\lambda'} - \varepsilon_\lambda\varepsilon_{\lambda'} - \Delta^2}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} \sim \left[1 - g\left(\frac{\varepsilon_\lambda - \varepsilon_{\lambda'}}{2\Delta}\right)\right]\delta(\varepsilon_\lambda + \varepsilon_{\lambda'}) \tag{9}$$

with :

$$g(x) = \frac{\sinh^{-1}(x)}{x\sqrt{1+x^2}}$$

The set of equations for ρ' is now :

$$\rho^{(1)}_{\lambda\lambda'} \sim \omega \ell_{\lambda\lambda'} \left[1 - g\left(\frac{e_\lambda - e_{\lambda'}}{2\Delta}\right)\right]\delta\left(\mu_F - \frac{e_\lambda + e_{\lambda'}}{2}\right) \tag{10}$$

$$\left\{\begin{array}{l} \rho^{(2)}_{\lambda\lambda'} \sim i\omega f_{\lambda\lambda'} \dfrac{e_\lambda - e_{\lambda'}}{2\Delta} \, g\left(\dfrac{e_\lambda - e_{\lambda'}}{2\Delta}\right)\delta\left(\mu_F - \dfrac{e_\lambda + e_{\lambda'}}{2}\right) \\ \displaystyle\sum_{\lambda\lambda'}\left[\frac{\hbar\dot{\ell}_{\lambda\lambda'}}{2\Delta} + f_{\lambda\lambda'}\left(\frac{e_\lambda - e_{\lambda'}}{2\Delta}\right)^2\right] g\left(\frac{e_\lambda - e_{\lambda'}}{2\Delta}\right)\delta\left(\mu_F - \frac{e_\lambda + e_{\lambda'}}{2}\right)\varphi_\lambda(\vec{r})\varphi^*_{\lambda'}(\vec{r}) = 0 \end{array}\right. \tag{11}$$

III - SEMICLASSICAL SOLUTION FOR A HARMONIC OSCILLATOR

The Wigner transform of the one-body normal density $\rho\,(\vec{r},\vec{p})$ can be given from eqs. (10-11) with the help of the moment of inertia :

$$\mathcal{J} = \frac{1}{\omega}\langle \ell_x \rangle = \frac{1}{\omega}\,\mathrm{Tr}(\ell_x \rho)$$

because in the Wigner-space the trace is :

$$\mathrm{Tr}\ A\,B = \int d^3\vec{r}\,\frac{d^3\vec{p}}{(2\pi\hbar)^3}\ A(\vec{r},\vec{p})\ B(\vec{r},\vec{p}) \tag{12}$$

for the product of two single-particle operators A and B. Taking into account the spins by the factor 2, the moment of inertia is given by :

$$\mathcal{J} = \frac{2}{\omega}\int d^3\vec{r}\,\frac{d^3\vec{p}}{(2\pi\hbar)^3}\ (\vec{r}\wedge\vec{p})_x\ \rho(\vec{r},\vec{p}) \tag{13}$$

To perform the change of representation for ρ : $\rho_{\lambda\lambda'} \longrightarrow \rho(\vec{r},\vec{p})$, the Fourier transform of g and δ in eqs (10-11) may be used :

$$g\left(\frac{e_\lambda - e_{\lambda'}}{2\Delta}\right) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} d\tau\ \exp\left[i\,\frac{e_\lambda - e_{\lambda'}}{2\Delta}\,\tau\right]\ g(\tau)$$

$$\delta\left(\mu_F - \frac{e_\lambda + e_{\lambda'}}{2}\right) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{+\infty} dt\ \exp\left[i\left(\mu_F - \frac{e_\lambda + e_{\lambda'}}{2}\right)t\right] \tag{14}$$

It is easy to show that the first term in $\rho_{\lambda\lambda'}^{(1)}$, which does not depend on the pairing field Δ ,gives exactly the rigid body contribution to the moment of inertia and does not contribute to the current (no classical current in the inner frame of a rigid body). Hence, omitting this first term in the following :

$$\rho_{\lambda\lambda'}^{(1)} = -\frac{\omega}{2\pi\sqrt{\hbar}}\int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} d\tau\ g(\tau)\, e^{i\frac{\mu_F t}{\hbar}} \langle\lambda| e^{-i\frac{Ht}{2\hbar}}\, e^{i\frac{H\tau}{2\Delta}}\, \ell_x\, e^{-i\frac{H\tau}{2\Delta}}\, e^{-i\frac{Ht}{2\hbar}} |\lambda'\rangle$$

In the Wigner-space, and to the order zero in $\hbar$, with $H_W = H_{classic}$:

$$\rho^{(1)}(\vec{r},\vec{p}) \sim -\frac{\omega}{2\pi\sqrt{\hbar}}\int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} d\tau\ g(\tau)\, e^{i\frac{\mu_F - H_{cl}}{\hbar}t}\ \left[\vec{r}(\tau)\wedge\vec{p}(\tau)\right]_x \tag{15}$$

with

$$F(\tau) = e^{i\frac{H}{\hbar}\frac{\hbar\tau}{2\Delta}}\ F(0)\ e^{-i\frac{H}{\hbar}\frac{\hbar\tau}{2\Delta}} \tag{16}$$

for any function F.
The integration over t is easily performed, but knowledge of the trajectory $\vec{r}(\tau)$ and then of the nuclear potential is necessary for the integration over τ . Proceeding in the same way for $\rho_{\lambda\lambda'}^{(2)}$ and $f_{\lambda\lambda'}$ leads to :

$$\rho^{(1)}(\vec{r},\vec{p}) = -\frac{\omega}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} d\tau\ g(\tau)\ \ell_x(\tau)\ \delta(\mu_F - H_{cl}) \tag{17}$$

$$\rho^{(2)}(\vec{r},\vec{p}) = \frac{\hbar\omega}{2\Delta\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\tau\, g(\tau)\, \dot{f}(\tau)\, \delta(\mu_F - H_{cl}) \tag{18}$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\tau\, g(\tau) \int \frac{d^3\vec{p}}{(2\pi\hbar)^3} \left[\frac{\hbar \dot{\ell}_x(\tau)}{2\Delta} - \frac{\hbar^2 \ddot{f}(\tau)}{4\Delta^2} \right] \delta(\mu_F - H_{cl}) = 0 \tag{19}$$

The present method may be applied to a harmonic oscillator, axially symetrical around the z-axis :

$$U(\vec{r}) = \frac{1}{2} m \left[\omega_y^2 (r_x^2 + r_y^2) + \omega_z^2 r_z^2 \right]$$

The trajectories are now, for each component (i = x,y,z)

$$r_i(\tau) = r_i \cos(\omega_i \frac{\hbar\tau}{2\Delta}) + \frac{p_i}{m\omega_i} \sin(\omega_i \frac{\hbar\tau}{2\Delta})$$

$$p_i(\tau) = p_i \cos(\omega_i \frac{\hbar\tau}{2\Delta}) - m\omega_i r_i \sin(\omega_i \frac{\hbar\tau}{2\Delta})$$

which gives $\ell_x(\tau)$, leading to :

$$\rho^{(1)}(\vec{r},\vec{p}) = -\omega \left(\frac{\omega_+ g_- - \omega_- g_+}{2\omega_z} r_y p_z - \frac{\omega_+ g_- + \omega_- g_+}{2\omega_y} r_z p_y \right) \delta(\mu_F - H_{cl}) \tag{20}$$

where $\omega_\pm = \omega_y \pm \omega_z$ and $g_\pm = g(\frac{\hbar\omega_\pm}{2\Delta})$

For the second term $\rho^{(2)}_{\lambda\lambda}$ it is necessary to know the function f and Migdal has shown that :

$$f(\vec{r}) = \alpha\, r_y r_z$$

is a solution of (19) defining :

$$\alpha = -2\Delta \frac{m}{\hbar} \omega_+ \omega_- \frac{g_+ + g_-}{\omega_+^2 g_+ + \omega_-^2 g_-}$$

then :

$$\rho^{(2)}(\vec{r},\vec{p}) = \frac{\hbar}{2m} \frac{d\omega}{\Delta} \left[\frac{\omega_+ g_+ - \omega_- g_-}{\omega_z} r_y p_z + \frac{\omega_+ g_+ + \omega_- g_-}{\omega_y} r_z p_y \right] \delta(\mu_F - H_{cl}) \tag{21}$$

Any mean-value can now be calculated from the Wigner transform $\rho'(\vec{r},\vec{p}) = \rho^{(1)}(\vec{r},\vec{p}) + \rho^{(2)}(\vec{r},\vec{p})$ of the density.

IV - SUPERFLUID CURRENTS IN AN ANISOTROPIC OSCILLATOR

In the body-fixed system of coordinates, the contribution to the current of the pairing field is :

$$\vec{j}^{(1)}(\vec{r})+\vec{j}^{(2)}(\vec{r}) = 2\int \frac{d^3\vec{p}}{(2\pi\hbar)^3}\,\frac{\vec{p}}{m}\left[\rho^{(1)}(\vec{r},\vec{p})+\rho^{(2)}(\vec{r},\vec{p})\right] \tag{22}$$

(the factor 2 takes the spins into account)
With :

$$2\int \frac{d^3\vec{p}}{(2\pi\hbar)^3}\,p_i^2\,\delta(\mu_F-H_{cl}) = m\,\rho_{TF}$$

$$\rho_{TF} = \frac{1}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}(\mu_F-V)^{3/2}$$

one finally finds :

$$\vec{j}^{(1)}\begin{cases} 0 \\ \omega\,\rho_{TF}\,\dfrac{\omega_+g_-+\omega_-g_+}{2\omega_y}\,r_z \\ -\omega\,\rho_{TF}\,\dfrac{\omega_+g_--\omega_-g_+}{2\omega_z}\,r_y \end{cases} \qquad \vec{j}^{(2)}\begin{cases} 0 \\ -\omega\rho_{TF}\,\dfrac{\omega_+\omega_-(g_++g_-)}{\omega_+^2g_++\omega_-^2g_-}\,\dfrac{\omega_+g_++\omega_-g_-}{\omega_y}\,r_z \\ -\omega\rho_{TF}\,\dfrac{\omega_+\omega_-(g_++g_-)}{\omega_+^2g_++\omega_-^2g_-}\,\dfrac{\omega_+g_+-\omega_-g_-}{\omega_z}\,r_y \end{cases} \tag{23}$$

for neutrons and protons separately.
Figure 1 shows the two limiting cases :
- the rigid body. The current in the laboratory frame is :

$$(\vec{j}_{rig})_{lab} = \rho\,\vec{\omega}\wedge\vec{r} = (0,-\omega\rho r_z,\omega\rho r_y)$$

and there is no current in the inner frame :

$$\vec{j}_{in} = \vec{j}_{lab} - \rho\,\vec{\omega}\wedge\vec{r} = 0$$

- the irrotationnal flow :

$$\vec{j}_{irrot} = -\gamma\,\vec{\nabla}(r_y r_z) = (0,-\gamma r_z,-\gamma r_y)$$

where γ is defined from the deformation

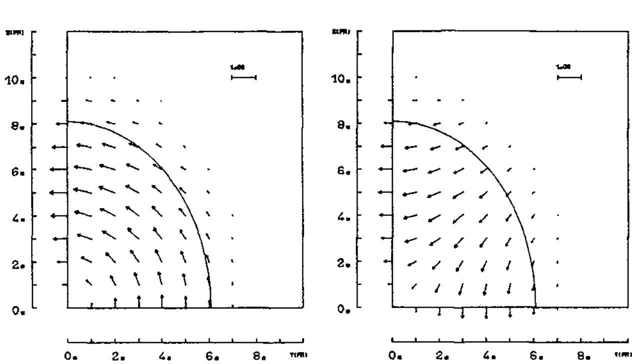


Fig.1

If the pairing potential vanishes, $g_{\pm} \to 0$ and, as already seen, the system behaves like a rigid body (which is the exact result of the Inglis cranking formula for a harmonic oscillator).
If the pairing potential greatly increases, $g_{\pm} \to 1$ and

$$\left[\vec{j}^{(1)} + \vec{j}^{(2)}\right]_{cm} = \left[0;\ \left(1 - 2\frac{\omega_y^2-\omega_z^2}{\omega_y^2+\omega_z^2}\right)\omega\rho_{TF}z;\ \left(-1-2\frac{\omega_y^2-\omega_z^2}{\omega_y^2+\omega_z^2}\right)\omega\rho_{TF}y\right].$$

or

$$\left[\vec{j}^{(1)} + \vec{j}^{(2)}\right]_{lab} = 2\,\frac{\omega_y^2-\omega_z^2}{\omega_y^2+\omega_z^2}\,(0\ ;\ -z\ ;\ -y)$$

and the flow is irrotational in the laboratory frame. This is in agreement with the value for the moment of inertia (Fig. 2). The asymptotic behaviour of $\mathfrak{J}$ (which goes to zero if only the usual Belyaev term is taken into account) is $\mathfrak{J}_{irrot}$ with the help of $\rho^{(2)}$(eq. 21).

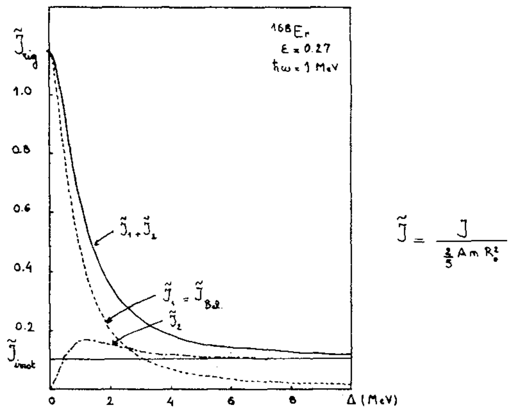


Fig. 2

Some flow patterns are drawn, for different values of the pairing potential Δ and for the nucleus ^{168}Er with

$$\hbar\omega = 1\ \text{MeV}$$

$$\hbar\,\omega_o = 41\ A^{-1/3}\ \text{MeV}$$

$$\hbar\,\omega_x = \hbar\,\omega_y = \hbar\omega_o\ \left(1+\frac{\varepsilon}{3}\right)$$

$$\hbar\,\omega_z = \hbar\omega_o\ \left(1-\frac{2\varepsilon}{3}\right)$$

$$\varepsilon = 0.272$$

The solid line represents an ellipse, with deformation ε with respect to a sphere $R_o = 1.2\ A^{1/3}$

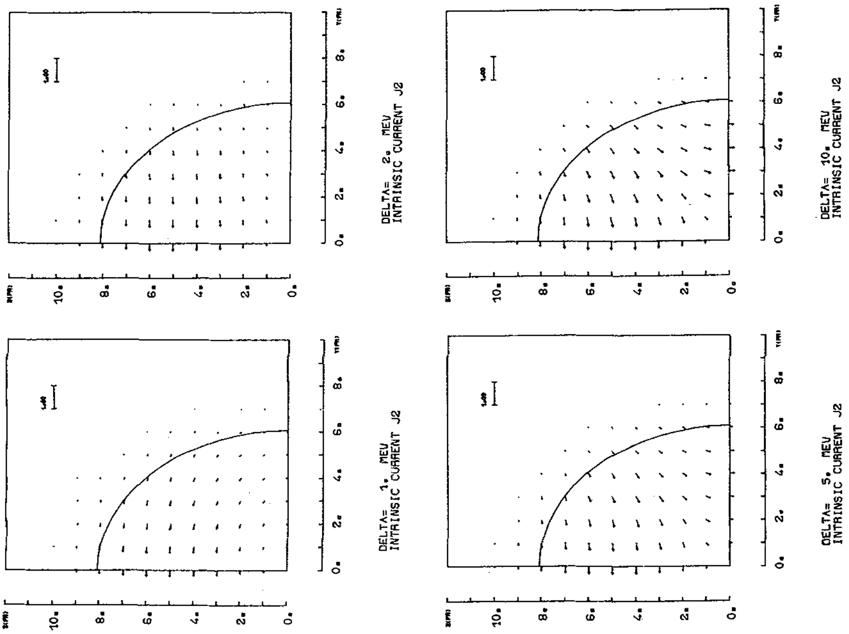

DELTA= 1. MEV
INTRINSIC CURRENT J2
DELTA= 2. MEV
INTRINSIC CURRENT J2
DELTA= 5. MEV
INTRINSIC CURRENT J2
DELTA= 10. MEV
INTRINSIC CURRENT J2

Fig. 4

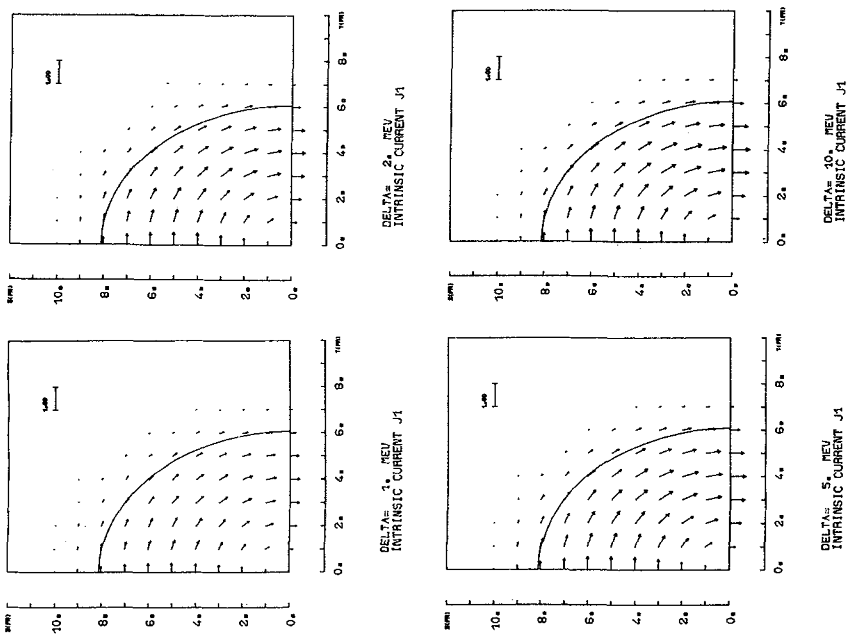
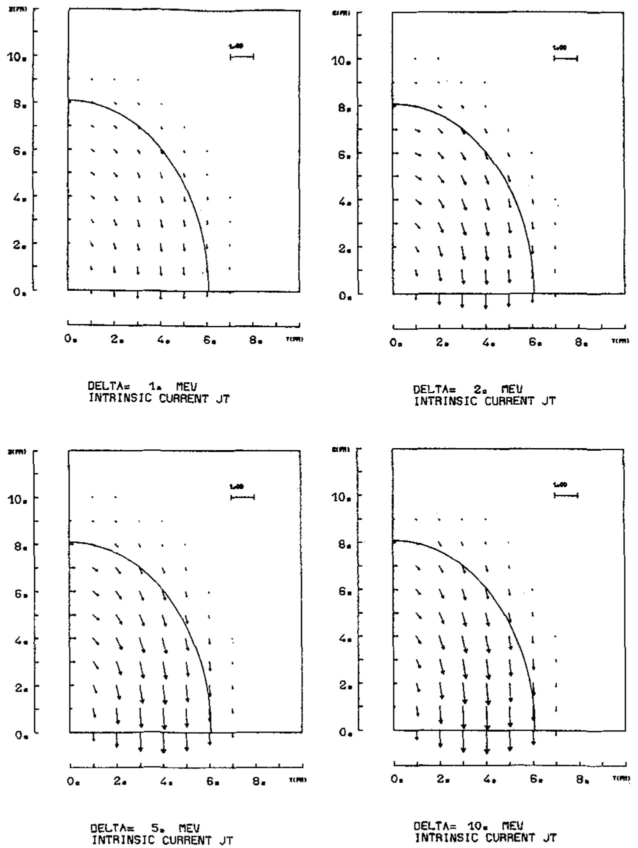

DELTA= 1. MEV
INTRINSIC CURRENT J1
DELTA= 2. MEV
INTRINSIC CURRENT J1
DELTA= 5. MEV
INTRINSIC CURRENT J1
DELTA= 10. MEV
INTRINSIC CURRENT J1

Fig.3


DELTA= 1. MEV
INTRINSIC CURRENT JT
DELTA= 2. MEV
INTRINSIC CURRENT JT
DELTA= 5. MEV
INTRINSIC CURRENT JT
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INTRINSIC CURRENT JT

Fig. 5

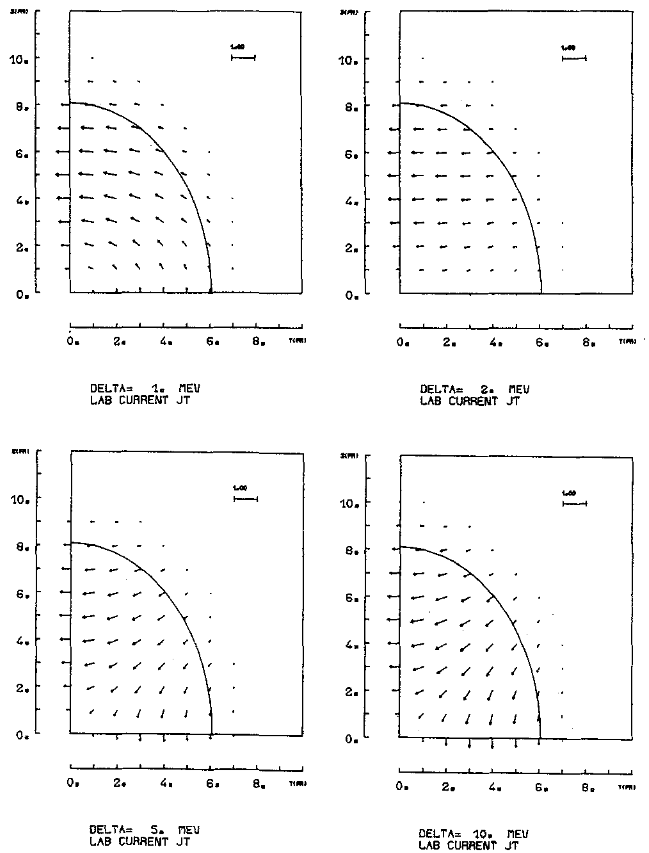
DELTA= 1. MEV
LAB CURRENT JT
DELTA= 2. MEV
LAB CURRENT JT
DELTA= 5. MEV
LAB CURRENT JT
DELTA= 10. MEV
LAB CURRENT JT

Fig. 6

Figures3 show the evolution of the intrinsic current $\vec{j}^{(2)}_{CM}$ due to the change in the pairing field in the body-fixed system : it changes clearly from a rigid body flow to an irrotationnal fluid flow.
The flow pattern for $\vec{j}^{(1)}_{cm}$ and $\vec{j}^{(2)}_{cm}$ are always very different, as shown in comparing figures 3 and 4. For low Δ values, the two currents flow in opposite ways, and for the highest Δ value they have different physical meanings : $\vec{j}^{(2)}$ is typically an irrotationnal fluid current and $\vec{j}^{(1)}$ is a rigid body rotation with velocity $(-\omega)$. That means that $\vec{j}^{(1)}$ alone would correspond to a case of no matter movement in the laboratory frame, although the potential is rotating. The total current $\vec{j}_{cm}^{(1)} + \vec{j}_{cm}^{(2)}$ is shown on fig. 5.
The net effect of the two corrections is a current $\vec{j}_{lab}$ in the laboratory frame (fig. 6) which evolves from a typical rigid body pattern to an irrotationnal fluid pattern, following the evolution of $\vec{j}^{(2)}_{cm}$. Thus, the introduction of the change Δ' of the pairing field by means of current $\vec{j}^{(2)}_{cm}$ is essential to reproduce the irrotationnal behaviour of a nucleus for large pairing fields.

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